# What is wrong in the current models of tunneling

Nikolay Chuprikov*[1]

[1]*Tomsk State Pedagogical University, Tomsk 634041, Russia*

In this paper we show that the conventional quantum-mechanical model of a non-resonant tunneling, as a subprocess of a one-dimensional completed scattering (OCS), is non-complete. It does not allow tracing the tunneling dynamics at all stages of scattering, what makes it impossible to resolve the tunneling time problem without coming into conflict with the (macro-)causality principle. As is shown, all timekeeping procedures to underlie the tunneling-time concepts and experiments presented in the tunneling time literature (TTL) fill this gap in the current description of the tunneling dynamics by 'self-evident' assumptions which are erroneous on closer inspection. We present the alternative model of the OCS, which allows tracing the tunneling dynamics at all stages of scattering and, as a consequence, is free from those paradoxes that flood the current TTL. By this model, among two fundamental velocity concepts of the wave dynamics – the flow velocity and the group one – only the former can be used to determine the velocity of tunneling particles in the barrier region.

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## I. INTRODUCTION

In the preprint [1] I first presented the idea that the only way to solve the long-standing tunneling-time problem (TTP) for a quantum particle scattering on a one-dimensional (1D) potential barrier (further this process will be shortly refereed to as the OCS) is to develop such a model of this process, which would allow tracing the tunneling dynamics at all stages of scattering. In [1] this model is developed for symmetric barriers, where the OCS is represented as a complex one consisting of two inseparable, coherently evolving subprocesses – transmission and reflection. As is shown, the total wave function $\Psi_{tot}$ to describe the OCS for a particle with a definite energy can be uniquely represented as the sum of two "subprocess wave functions" $\psi_{tr}$ and $\psi_{ref}$, each having one outgoing wave and one incoming wave joined 'causally' at the midpoint $x_c$ of the barrier region – at this point each subprocess wave function is continuous together with the corresponding probability current density.

This approach does not lead to superluminal tunneling velocities. However, excepting the papers [2–4], all other attempts to bring up this approach for a wide and open discussion have faced with a very tough resistance of reviewers. To show what is unconvincing in my works for experts on the TTP, it is worth quoting here the referee comment on my recent article [5] where I argue, in particular, that in the *tunneling time* literature (TTL) neither the Buttiker dwell time nor the Wigner phase time, nor other contenders on the role of the *tunneling time* play really this function.

This negative assessment of the existing tunneling time concepts has been perceived in this comment as a blasphemy and result of my incompetence in the matter:

> "I . . . cast serious doubt on the technical validity of the work presented. In particular the decomposition of the wave function into a transmitted and reflected portion at a point $x_c$ is not proven. . . While the tunneling time literature has as many views as there are authors the dwell time is one of the few time scales on which the community has shown wide agreement. The author calls even the dwell time into question . . . The literature is poorly represented. 1) Local Larmor times have been discussed in a number of papers. . . [see [6, 7]] 2) Subluminal traversal times have been discussed in a number of works: [see [8, 9]] . . . The material presented by the author is neither technically nor in scope on a level that is appropriate for a . . . journal".

Of course, one cannot prove or disprove anything by means of such arguments as "serious doubt" and "wide agreement". At the same time, I see now that I really need to make further efforts in order to dispel any doubt of experts on the significance of my approach as well as to dispel the widespread myth on the technical validity of the "tunneling-time" concepts presented in the current TTL.

Indeed, what does it mean "the decomposition of the wave function into a transmitted and reflected portion at

* E-mail: chnl@tspu.edu.ru

a point $x_c$ is not proven"? The case is that, from the mathematical point of view, the presented formalism (for details, see Section III A) to provide the unique decomposition of the total wave function into the subprocess wave functions is surely proven in [1–4]. As regards quantum mechanics (QM), the technique of decomposition of the OCS into alternative subprocesses is beyond the current practice of this theory. It does not imply and does not forbid this decomposition.

In this connection, I see that the proof of the significance and validity of this decomposition should contain the following two steps: (a) the proof of the current description of the OCS as being inconsistent on the macro-scales with the causality principle; (b) the proof of the fact that this decomposition technique makes the quantum-mechanical description of the OCS compatible not only with special relativity but also with classical mechanics, with respecting the principles of QM itself.

The first step is realized in Section II where I critically analyze the current TTL and show that all the most prominent tunneling-time concepts and experiments, including those pointed out in [6–9], are based on (explicit or implicit) 'self-evident' assumptions which are erroneous on closer inspection, what makes it impossible to unambiguously interpret these concepts and experiments. On the second step (Section III) I present the key points of my approach [1–4] and show that the idea of decomposing the OCS into alternative subprocesses makes the quantum-mechanical description of this process compatible with classical probability theory as well as with the (macro-)causality principle.

## II. CRITICAL ANALYSIS OF THE CURRENT TTL

### A. On the criteria used for selecting the approaches in the TTL for the detailed analysis

To select the most significant approaches and tunneling time concepts for the detailed analysis, I will proceed from the following considerations:

(a) The TTP appears not only in QM, but also in classical electrodynamics (CED). In both cases the TTP is associated with the wave dynamics (it is worth to recall here that the alternative name of QM is "wave mechanics"), thus the way of solving this problem must be common for both theories. The key idea of this way must be independent on the nature of scattering waves. That is, whether the time in QM is a parameter or physical observable, this thing must be unimportant in solving the TTP. From this point of view, this problem does not need introduction of the time operator into QM. Therefore all approaches to imply introducing this operator will not be considered here. The only exception will be made for the time-operator procedure [10] which, by its authors, is extendable onto quantum electrodynamics (see Section II E).

(b) The wave-packet approach is the only one which is inherent to both theories – QM and CED. Thus, contrary to Landauer and Martin [11], the main contenders in QM on the role of the tunneling time should be searched for among the concepts introduced within this approach and its clock-based versions. The Feynman path-integral approach to the TTP as well as that based on the Wigner function will not be considered here as those laying beyond the conventional quantum-mechanical description of the OCS. As regards the Bohmian model of this process, in fact it is a visualization of the conventional model of the OCS, and thus its analysis is of interest here.

(c) The TTP should be solved first within the standard setting of the OCS. Therefore those approaches to study the bilateral scattering as well as settings with the cutoff initial conditions will not be considered here.

(d) There is a simple case in the wave-packet analysis when the description of the temporal aspects of the wave dynamics meets no problems. This takes place when waves or wave packets move freely in the infinite uniform space. For this purpose, the wave-packet approach has at its disposal two basic concepts of the wave velocity – the (probability or energy) flow velocity and the group velocity. For the free wave dynamics these concepts agree with each other. Thus the TTP should be resolved first on the basis of these two concepts. The fact that these concepts lead to superluminal tunneling velocities within the existing approaches discredits these approaches, rather than the concepts themselves.

### B. Standard model of the OCS

Let a particle impinge from the left the potential barrier $V(x)$ confined to the spatial interval $[a, b]$; $d = b - a$ is the barrier width; $x_c = (b + a)/2$ is the midpoint of the barrier region. Following the alternative approach [2–4] we will suppose the barrier to be symmetric: $V(x_c - x) = V(x - x_c)$. The wave function $\Psi_{tot}$ for a

particle with a given energy $E = \hbar^2 k^2/2m$ reads in this case as follows:

$$\Psi_{tot}(x;k) = \begin{cases} e^{ikx} + b_{out}(k)e^{ik(2a-x)} & x < a \\ a_{tot}F(x-x_c;k) + b_{tot}G(x-x_c;k) & a < x < b \\ a_{out}(k)e^{ik(x-d)} & x > b \end{cases} \tag{1}$$

$F(x;k)$ and $G(x;k)$ are such real partial solutions to the Schrödinger equation in the interval $[a,b]$ that $F(x_c - x;k) = -F(x-x_c;k)$, $G(x_c - x;k) = G(x-x_c;k)$; the ($x$-independent) Wronskian $\frac{dF}{dx}G - \frac{dG}{dx}F$ will be denoted here via $\kappa$;

$$a_{out} = \frac{1}{2}\left(\frac{Q}{Q^*} - \frac{P}{P^*}\right) \equiv \sqrt{T(k)}\exp(iJ(k)), \quad b_{out} = -\frac{1}{2}\left(\frac{Q}{Q^*} + \frac{P}{P^*}\right) \equiv \sqrt{R(k)}\exp\left[i\left(J(k) - \frac{\pi}{2}\right)\right];$$

$$a_{tot} = -\frac{1}{\kappa}P^* a_{out}e^{ika}; \quad b_{tot} = \frac{1}{\kappa}Q^* a_{out}e^{ika};$$

$$Q = \left[\frac{dF(x-x_c;k)}{dx} + ikF(x-x_c;k)\right]_{x=b}; \;\; P = \left[\frac{dG(x-x_c;k)}{dx} + ikG(x-x_c;k)\right]_{x=b};$$

here $T(k)$ and $R(k)$ are the transmission and reflection coefficients, respectively; $T(k) + R(k) = 1$. If $V(x)$ is the rectangular barrier of height $V_0$ and $E < V_0$, then

$$F = \sinh(\kappa x), \quad G = \cosh(\kappa x), \quad \kappa \equiv \frac{dF}{dx}G - \frac{dG}{dx}F = \sqrt{2m(V_0 - E)}/\hbar.$$

In the non-stationary case the OCS is described by the wave packet

$$\Psi_{tot}(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathcal{A}(k)\Psi_{tot}(x;k)e^{-iE(k)t/\hbar}dk, \tag{2}$$

where $\mathcal{A}(k)$ is, for example, the Gaussian function: $\mathcal{A}(k) = (2l_0^2/\pi)^{1/4}\exp(-l_0^2(k-k_0)^2)$. In this setting the problem, the wave function represents at the initial time $t = 0$ the wave packet of width $l_0$ ($l_0 \ll a$), with the center of "mass" (CM) at the point $x = 0$.

The incident $\Psi_{inc}(x,t)$, transmitted $\Psi_{tr}(x,t)$ and reflected $\Psi_{tr}(x,t)$ wave packets have the forms

$$\Psi_{inc}(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathcal{A}(k)e^{i[kx - iE(k)t]/\hbar}dk$$

$$\Psi_{tr}(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathcal{A}(k)\sqrt{T(k)}\exp\left[i\left(J(k) + k(x-d) - E(k)t/\hbar\right)\right] \tag{3}$$

$$\Psi_{ref}(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\mathcal{A}(k)\sqrt{R(k)}\exp\left[i\left(J(k) - \frac{\pi}{2} + k(2ka - x) - E(k)t/\hbar\right)\right];$$

note that $T(-k) = T(k)$ and $J(-k) = -J(k)$.

### C. The dwell time

Let us begin our analysis of the tunneling time concepts with the dwell time. Unlike the group time it can be introduced both for single waves and for wave packets. In QM the dwell time is defined through the velocity to characterize the probability flow density. This fundamental concept of the wave velocity will be referred to as the wave flow velocity.

To fix the intrinsic properties of the dwell time, we first consider the case when the interval $[a,b]$ is empty, i.e., when $V(x) \equiv 0$ in this interval. The ensemble of free particles is described by the wave function $\Psi_{tot}(x;k) = \Psi_{free}(x;k) = e^{ikx}$. The velocity $v_{flow}^{free}$ of particles in this ensemble is introduced via the (probability) flow

density $I_{free}$, which can be written as $I_{free} = |\Psi_{free}(x;k)|^2 v_{flow}^{free}$. Thus, for this velocity and the average time $\tau_{dwell}^{free}$ spent by a particle in the interval $[a,b]$, we have

$$v_{flow}^{free} = \frac{I_{free}}{|\Psi_{free}(x;k)|^2} = \frac{\hbar k}{m}, \qquad \tau_{dwell}^{free} = \frac{1}{I_{free}} \int_a^b |\Psi_{free}(x;k)|^2 dx = \frac{md}{\hbar k}. \tag{4}$$

As is seen from (4), for a free particle the wave flow velocity $v_{flow}^{free}$ is the transit velocity, and the dwell time $\tau_{dwell}^{free}$ is the transit time.

Let us now consider the OCS, i.e., the potential $V(x)$ is now nonzero. The wave function to describe this process is given by Exps. (1), and the corresponding flow density is $I_{tot} = I_{tr} = T(k)\hbar k/m$. Then, acting as in the previous case, for the wave flow velocity and the dwell time we obtain

$$v_{flow}^{(1)} = \frac{I_{tot}}{|\Psi_{tot}(x;k)|^2}, \qquad \tau_{dwell}^{(1)} = \frac{1}{I_{tot}} \int_a^b |\Psi_{tot}(x;k)|^2 dx. \tag{5}$$

The concept of the dwell time $\tau_{dwell}^{(1)}$ is widely spread in CED (see, e.g., [12–14]). However, the physical meaning of this quantity is unclear. The point is that unlike the wave function $\Psi_{tot}(x;k)$ the corresponding flow density $I_{tot}(x;k)$ is associated only with transmitted particles. Due to this paradoxical property of the wave function $\Psi_{tot}(x;k)$ to describe the OCS (see also Sections II L and V) the quantities $v_{flow}^{(1)}$ and $\tau_{dwell}^{(1)}$ characterize neither the whole ensemble of scattering particles nor its transmitted part.

In this connection, of widely spread in QM is the variant of the dwell time introduced by Buttiker (see [15] and Section II G):

$$v_{flow}^{(2)} = \frac{I_{inc}}{|\Psi_{tot}(x;k)|^2}, \qquad \tau_{dwell}^{(2)} = \frac{1}{I_{inc}} \int_a^b |\Psi_{tot}(x;k)|^2 dx, \tag{6}$$

where $I_{inc} = \hbar k/m$. The flow density $I_{inc}$ like the density $|\Psi_{tot}(x;k)|^2$ describes the whole ensemble of particles. But now we meet with another problem – the substitution $I_{inc}$ for $I_{tot}$ is made here 'by hand' because $I_{inc}$ is not linked to $|\Psi_{tot}(x;k)|^2$ in the barrier region $[a,b]$.

Thus, unlike $\tau_{dwell}^{free}$, the time scales $\tau_{dwell}^{(1)}$ and $\tau_{dwell}^{(2)}$ have an unclear physical meaning. None of them can be unambiguously ascribed to the whole ensemble of particles or to its transmitted portion. In particular, for $E < V_0$ both these quantities are, strictly speaking, neither the tunneling time nor the scattering time to describe the whole scattering process. Of course, all these statements are valid when $R(k) \neq 0$. If $R(k) = 0$, $\tau_{dwell}^{(1)} = \tau_{dwell}^{(2)}$, and both these coinciding quantities give the transit time.

### D. The group time

The group velocity and time are defined for wave packets. Following the Wigner phase time concept, in this section we shall consider the wave-packet dynamics in the limiting case $l_0 \to \infty$, assuming that at the initial time $t = 0$ the wave packet peaks in the $k$-space at the point $k$. As in the previous case, we begin with the free motion. In this case $\Psi_{tot}(x,t)$ coincides with $\Psi_{inc}(x,t)$ (3) extrapolated onto the whole $OX$-axis. The position $<\hat{x}>_{free}$ of the CM of this wave packet, for any value of $t$, is determined by the expression $<\hat{x}>_{free} = \hbar k t/m$. Thus, for the time $\tau_{group}^{free}$ spent by the CM in the region $[a,b]$ we have $\tau_{group}^{free} = md/\hbar k$. That is, in this case the group time $\tau_{group}^{free}$ coincides with the dwell time $\tau_{dwell}^{free}$ (4); both give the transit time to characterize the free wave dynamics in the region $[a,b]$.

The situation changes drastically in attempting to introduce the group time for the OCS. The first problem to appear in this case is that for a non-resonant transmission the transmitted portion of the scattering wave packet is seen only at the final stage of scattering, i.e., far from the barrier in the transmission region. This means that, within the standard model of the OCS, the *group tunneling* time can be introduced, in the best case, only for some asymptotically large spatial region, e.g., for the interval $[0, b+L]$ where $L \gg l_0$. That is, in the best case, it might be defined as an *asymptotic* time scale.

However, even this cannot be done properly within the conventional approach. Indeed, this approach allows us to define correctly the time of arrival $\tau_{tr}^{ar}(x)$ of the CM of the transmitted wave packet at the distant point $x$ behind the barrier. For this purpose we can use the explicit expression for the position $<\hat{x}>_{tr}(t)$ of the CM

of the transmitted wave packet,

$$< \hat{x} >_{tr} (t) = \frac{\hbar k t}{m} - J'(k) + d;$$

here the prime denotes the derivative on $k$. "However, before the arrival time is related to a "transit time" one must know the departure time of the thing that arrived" [16]. But namely this step is performed erroneously in the existing approaches. Without further ado, it is assumed as self-evident that the departure time of transmitted particles coincides with that of the incident wave packet (which is zero in the considered setting of the problem). As a result, by the conventional approach, in this setting the arrival time

$$\tau_{tr}^{ar}(b+L) = \frac{m}{\hbar k}\left(J'(k) + L + a\right)$$

gives the group transit time for the interval $[0, b+L]$. Without the contributions of the outer regions $[0, a]$ and $[b, b+L]$, this expression yields the Wigner time $\tau_{ph}$ and ("phase") delay time $\tau_{del}$:

$$\tau_{ph}(k) = \frac{m}{\hbar k} J'(k) \qquad \tau_{del}(k) \equiv \tau_{ph} - \tau_{group}^{free} = \frac{m}{\hbar k}\left[J'(k) - d\right]. \tag{7}$$

By Hauge and Støvneng [17], $\tau_{ph}$ is the asymptotic (extrapolated) average tunneling time which is associated with the quantum dynamics of a particle in the asymptotically large interval $[0, b+L]$, rather than with the barrier region $[a, b]$; correspondingly, $\tau_{del}$ is the difference between the time of arrival at the distant point of the CM of the transmitted wave packet and the time of arrival at this point of the reference wave packet that moves freely at all stages of scattering and coincides at the initial time with the incident wave packet.

However, the Wigner time is very often treated as the average time spent by a tunneling particle precisely in the barrier region (see, e.g., [18, 19]. As is said in [19], "The Wigner time accounts for how long it takes for the peak of a wave packet to emerge from the exit face of the tunnel barrier relative to the time that the peak of the incident wave packet arrived at the entrance face". Or else, "A parameter commonly used to estimate the time such a particle spends in the barrier region is the phase time $\tau_{ph}$, essentially the energy derivative of the phase of the transmission amplitude" [20].

According to this interpretation, the saturation of the phase tunneling time in the opaque limit – the usual [21] and generalized [10, 18] (see also [22]) Hartman effects – means that the group tunneling velocity in the barrier region can be superluminal. However, as was said above, when $R(k) \neq 0$ the timekeeping procedure to underlie this concept violates the causality principle at the *asymptotically large* scales, because the arrival time and departure time used in this procedure are defined for two events which are causally disconnected with each other (see [23]). Thus, the Wigner time $\tau_{ph}$ and respective delay time $\tau_{del}$ have been introduced within the incorrect timekeeping procedure, and hence no physical meaning can be ascribed to these quantities.

### E. The "non-coherent flux-separation" procedure

By Nussenzveig the TTP is an ill-posed problem – the definition of the tunneling time on the basis of such fundamental concepts as the flow velocity and group velocity is impossible in principle because "transmission and refection are inextricably intertwined" [24]. This pessimistic view on the possibility of a successful introduction of individual characteristic times for tunneling (transmission) has been repeatedly appeared in the TTP (see, e.g., [16, 17, 25]).

In this connection, it is worthwhile to consider the timekeeping procedure presented in [10] whose authors claim that their approach based on "the non-coherent flux-separation" technique allows the individual description of transmission and reflection in the spatial regions asymptotically remote from the barrier region.

This claim is erroneous on closer inspection. Firstly, the time operators $\hat{t}_{\pm}(x)$ introduced in this approach imply averaging over time, what is alien to QM. Secondly, the authors claim that the formalism of these operators agrees with the conventional QM. However, in fact, this is proven only for the initial and final stages of the time-dependent OCS, i.e., for the asymptotically remote (from the barrier) spatial regions, when the total flux $J$ is equal either to $J_+$ or $J_-$; $J_\pm = J\Theta(\pm J)$ (see p.137); here the positive $J_+$ and negative $J_-$ fluxes describe forward and backward motion, respectively. Thirdly, even if this formalism would be compatible with QM in all spatial regions, it could not overcome those difficulties to appear within the existing group tunneling-time concept.

Indeed, the probabilities $\rho_>(x_i, t)$ and $\rho_>(x_f, t)$ defined for the remote points $x_i$ and $x_f$ to lie on the different sides of the barrier describe the one-particle ensembles between whom there is no causal relationship. The time

scale $\langle \tau_T(x_i, x_f) \rangle = \langle t_+(x_f) \rangle - \langle t_+(x_i) \rangle$, defined as the "differences between the mean times referring to the passage of the final and initial wavepackets through the relevant space-points", in fact bears no relation to dwelling the subensemble of *tunneling* particles in the region $[x_i, x_f]$. In this approach the departure time $\langle t_+(x_i) \rangle$ does not describe the subensemble of *to-be-transmitted* particles whose time of arrival at the point $x_f$ is described by $\langle t_+(x_f) \rangle$. Thus, according to our analysis, the "non-coherent flux-separation procedure" like the Wigner tunneling-time concept violates the (macro-)causality principle!

### F. Total and partial densities of states

In this section we consider the timekeeping procedure [6, 7] to define the characteristic times of the OCS through the partial densities of states (PDOSs). These quantities appear within the scattering-matrix formalism to describe the response of the system under study to the infinitesimal variation of the potential $V(x)$. This approach is of interest because, as is claimed in [6], PDOSs carry the information not only about the future of scattering particles, but also about their past.

Let the OCS be characterized by a scattering matrix with elements $S_{\alpha\beta}$, where the indices $\alpha$ and $\beta$ label, respectively, outgoing and incoming scattering channels of the system under study (see [6]). The local PDOS $d\eta_{\alpha\beta}/dE$ are written in [6] in the form

$$\frac{d\eta_{\alpha\beta}}{dE}(x) \equiv -\frac{1}{4\pi i}\left( S^{\dagger}_{\alpha\beta}\frac{\delta S_{\alpha\beta}}{\delta V(x)} - \frac{\delta S^{\dagger}_{\alpha\beta}}{\delta V(x)} S_{\alpha\beta}\right) \tag{8}$$

where the off-diagonal PDOSs are always positive; $\delta/\delta V(x)$ denotes a functional derivative.

Then the injectivity $d\eta^{inj}_{\beta}/dE$ of the incoming channel $\beta$ as well as the emissivity $d\eta^{emis}_{\alpha}/dE$ into the outgoing channel $\alpha$ are

$$\frac{d\eta^{inj}_{\beta}}{dE}(x) = \sum_{\alpha}\frac{d\eta_{\alpha\beta}}{dE}(x), \qquad \frac{d\eta^{emis}_{\alpha}}{dE}(x) = \sum_{\beta}\frac{d\eta_{\alpha\beta}}{dE}(x). \tag{9}$$

The PDOSs, injectivity and emissivity enter into the decomposition of the total DOS as follows

$$\frac{d\eta}{dE}(x) = \sum_{\alpha\beta}\frac{d\eta_{\alpha\beta}}{dE}(x) = \sum_{\beta}\frac{d\eta^{inj}_{\beta}}{dE}(x) = \sum_{\alpha}\frac{d\eta^{emis}_{\alpha}}{dE}(x). \tag{10}$$

As was said in [6] about PDOSs, "They are based on both a preselection and postselection of carriers, i.e., they group carriers according to the asymptotic region from which they arrive ($\beta$) and according to the asymptotic region into which they are scattered ($\alpha$). We emphasize that the PDOSs are mathematical constructions. Whether these quantities are by themselves of physical relevance might well depend on the problem under investigation."

Note that in the case of the OCS, for scattering channels located at the left and right sides of the barrier region, we have $\alpha, \beta = 1$ and $\alpha, \beta = 2$, respectively (see [6]). Thus, when a particle impinges on the barrier from the left, the relevant local PDOSs are $d\eta_{11}/dE$ and $d\eta_{21}/dE$, respectively.

As was shown in [6], the corresponding injectivity $d\eta^{inj}_{1}/dE$ determines the time scale $d\tau_1$ is

$$d\tau_1(x) = \frac{|\Psi_{tot}(x)|^2}{I_{inc}}dx = 2\pi\hbar\frac{d\eta^{inj}_{1}}{dE}(x)dx. \tag{11}$$

That is, $d\tau_1/dx$ coincides with the dwell time $\tau^{(2)}_{dwell}$ discussed in Section II C. Besides, in some cases the local PDOSs are connected to the local Larmor times (see expressions (57–60) in [6]) which are considered in [6] as "physically well-defined quantities" to describe the OCS. By the authors of the paper, "The results (57)–(60) connect the local PDOS with physically well-defined quantities, which indicates the relevance of the PDOS".

However, this is not. The physical relevance of the PDOSs in studying the temporal aspects of the OCS is moot. Firstly, the PDOSs $d\eta_{21}/dE$ and $d\eta_{11}/dE$ to enter Exps. (57–60) connect the outgoing channels $\beta = 2$ and $\beta = 1$ with *the same incoming channel* $\alpha = 1$. This means that none of these outgoing channels is linked causally to this incoming channel. Incoming channels that would be causally linked to either of these two outgoing channels are unknown within the conventional model of the OCS (it is this problem that

is under study in our approach [1–4] (see Section III A)). So that the PDOSs $d\eta_{21}/dE$ and $d\eta_{11}/dE$ have no relation to transmission and reflection. Secondly, the Larmor-clock procedure does not work properly within the conventional model. Revealing the main shortcomings of the existing Larmor-clock procedure is our next goal.

### G. Clock-based timekeeping procedures

Initially proposed in the works [26], this procedure was developed further by Buttiker [15]. Its main idea is as follows. An infinitesimal magnetic field directed along the $OZ$-axis is confined to the barrier region $[a, b]$ on the $OX$-axis. At the initial time $t = 0$ a beam of electrons scattered by the potential barrier is in the quantum state to represent the statistical mixture of two subensembles of particles with the $z$-th spin components $\hbar/2$ and $-\hbar/2$. This state is assumed to be such that the electron spin averaged over the mixture is strictly orthogonal at $t = 0$ to the magnetic field and the direction of the motion of particles. Outside the barrier the spin is constant. When electrons enters the barrier region the average spin starts a Larmor precession. When they leave the barrier the precession stops.

In this timekeeping procedure the average spin of particles plays the role of a clockwise. For transmitted particles the final position of the clockwise coincides with the direction of the electron spin averaged over the transmitted portion of the scattered beam. Its initial position coincides, as is assumed in [15], with the direction of the spin averaged over the *whole incident* beam, which, as is said above, is strictly orthogonal to the magnetic field and the velocity of particles.

As is shown in [15], in the course of scattering the average spin of transmitted particles not only rotates due to the Larmor precession in the plane orthogonal to the magnetic field, but also acquires a nonzero $z$-th component. As a consequence, the Larmor timekeeping procedure provides two independent characteristic times for transmission: $\tau_{\perp}^{tr}$ associated with the Larmor precession, and $\tau_{\parallel}^{tr}$ associated with the emergence of a spin component parallel to the magnetic field (see also Section III D). That is, figuratively speaking, there are in fact two Larmor clocks associated with transmission. By Buttiker the Larmor time $\tau_{\perp}^{tr}$ is precisely the dwell time $\tau_{dwell}^{(2)}$, and $\tau_{\parallel}^{tr}$ determines the so-called traversal time which coincides in the opaque limit with the Büttiker-Landauer time [23].

Note that the effect of aligning the average spin with the magnetic field was found for reflected particles too. As was shown in [15], the corresponding Larmor time $\tau_{\parallel}^{ref}$ is such that $T\tau_{\parallel}^{tr} + R\tau_{\parallel}^{ref} = 0$ for any given energy $E$.

However, the equality $\tau_{\perp}^{tr} = \tau_{dwell}^{(2)}$ is paradoxical in essence. Indeed, it says that the dwell time $\tau_{dwell}^{(2)}$ defined in terms of the total wave packet to move in the barrier region (see Section II C) turns out to coincide with the time scale $\tau_{\perp}^{tr}$ defined in terms of the transmitted wave packet to move far from the barrier region.

The emergence of the nonzero scattering times $\tau_{\parallel}^{tr}$ and $\tau_{\parallel}^{ref}$ is even more paradoxical. The point is that, according to the assumption made in [15], the $z$-th component of the average spin for both subprocesses is zero at the initial time. Thus, this spin component, as a motion integral in this scattering problem, must be zero at all stages of scattering. At the same time the Larmor procedure violates this requirement. Moreover, this "effect" appears for the reflection subprocess even when the infinitesimal magnetic field is switched on far from the barrier region in the transmission zone. As was said in [27, 28] in this connection, "the Larmor-clock approach leads to a result contrary to the common sense notion that a reflected particle does not spend any time on the far side . . . of the potential barrier". (As is shown in [3] (see also Section III D), the $z$ components of the average spin, for transmission and reflection, are *nonzero* at the initial time and the "interactions" times $\tau_{\parallel}^{tr}$ and $\tau_{\parallel}^{ref}$ are, in fact, the initial positions of the Larmor clocks for transmission and reflection, respectively. These quantities remain constant in the course of scattering and, thus, do not measure the duration of these subprocesses.)

These results cannot be considered as well-established – there are two 'self-evident' assumptions made in this Larmor-clock procedure, which undermine their legitimacy. The first one is that "The polarization of the transmitted (and reflected) particles is compared with the polarization of the incident particles" [15]. This step is evident to contradict the observation [23] that there is no causal relationship between the transmitted (reflected) and incident particles.

Another 'self-evident' assumption concerns the dynamics of the average spin of transmitted particles in the plane orthogonal to the magnetic field. As is assumed in [15], in the barrier region the spin experiences only the (smooth) Larmor precession in this plane. But this assumption is justified only for the spin averaged over

the *whole* beam of particles, whose state experiences the unitary quantum evolution at all stages of scattering. At the same time, transmission is only a part of the OCS. And, thus, if the dynamics of this subprocess was everywhere unitary it could take place separately from reflection. But, this is not – in the case of a non-resonant tunneling of a particle through a semi-transparent potential barrier, the subprocesses of the OCS cannot occur separately from each other.

Thus, the implicit assumption made in [15] about the unitarity of the tunneling dynamics in the barrier region is certainly false, and one should not exclude that, apart from the Larmor precession, there are other physical effects to alter the average spin of transmitted particles in the barrier region (see Section III D). To clearly answer this question, one has to reveal the dynamics of transmitted particles at all stages of scattering.

All this concerns also the clock-based timekeeping procedures [22] and [23]. The former is similar, in every respect, to the Larmor-clock procedure (both the Salecker-Wigner-Peres procedure [22] and Larmor one deal with the ensembles of particles at the initial and final stages of scattering, which are not linked causally to each other). As regards the Büttiker-Landauer time [23], it coincides in the limit $k \to 0$ with the Larmor time $\tau_{\parallel}^{tr}$ which actually has no concern to the tunneling time (see also [17]).

### H. "Tunneling Confronts Special Relativity" [33]: Why this claim cannot be proved or disproved within the current model of a non-resonant tunneling

So, neither the Larmor time $\tau_{\perp}^{tr}$ (and thus Buttiker's dwell time $\tau_{dwell}^{(2)}$) nor the Wigner group time $\tau_{ph}$ can be considered as well established tunneling time concepts. None of them allows a correct definition of the tunneling *velocity* (see also [16, 17]). At the same time many authors claim they measure the tunneling velocity and even observe *superluminal* tunneling velocities predicted by these concepts (see, e.g., [29–40]). All this has led to a deep controversy in the TTL. Some researches (see, e.g., [33]) believe that "tunneling confronts special relativity", while others claim that such velocities result from the so-called reshaping process that respects the causality principle (see, e.g., [8, 20, 30, 32, 41–43]).

Winful [16] is one of few investigators who considers the above claims as untenable. In his excellent critical analysis [16] he proves that the existing tunneling velocities cannot be treated unambiguously as the transit velocity, and hence it is meaningless to say about *measuring* the superluminal or subluminal tunneling (transit) velocities. However, Winful's critique has been ignored in fact. At present the so called "reshaping argument" as well as the signal-velocity and dispersion-relations arguments are regarded as more weighty evidence than those presented by Winful. But is it really? Let us consider this question in detail.

I think that the quintessence of the "reshaping argument" (see, e.g., [8, 20, 30, 32, 41–43]) put forward for explaining the Hartman effect has been expressed in the most concise and clear manner in the following quotations taken from the work [30]:

> "In classical optics, the existence of group velocities greater than $c$, and even negative ones under certain conditions, is known, and has been observed experimentally. . . This phenomenon is understood as a "pulse reshaping" process, in which a medium preferentially attenuates the later parts of an incident pulse, in such a way that the output peak appears shifted towards earlier times. . . the group delay (or "phase time") gives a better description of the physically observable delay than does the "semiclassical" interaction time."

> "Although the apparent tunneling velocity $(1.7 \pm 0.2)c$ is superluminal, this is not a genuine signal velocity, and Einstein causality is not violated."

Having been developed twenty years ago, this explanation of the Hartman effect (or, at least, its ingredients such as the signal-velocity and dispersion-relations arguments) remains popular to this day (see, e.g., [35–39]). Since this explanation is moot in reality, of importance is to critically analyze this quotation in detail.

**(i) The group delay (or "phase time") gives a better description of the physically observable delay than does the "semiclassical" interaction time:**

However, this is not surprising because the "single-photon" tunneling-time experiments [30, 35, 36] follow exactly the timekeeping procedure to underlie the theoretical concept of the group delay time (see also [16, 41]). Of importance is also to stress that these experiments like this concept deal eventually with single-photon ensembles rather than with single photons.

**(ii) The existence of group velocities greater than $c$. . . has been observed experimentally:**

This is not surprising for the same reason – the same timekeeping procedure to violate the causality principle, the same nonphysical result. I have to stress once more (see also [16]) – in the tunneling time experiments and group tunneling-time concept, the departure time is defined for the photon ensemble which is causally disconnected with the subensemble of transmitted photons. Another, more serious shortcoming of this procedure is the implicit assumption that the *group transmission* velocity – the velocity of the CM of the wave packet to describe the transmission subprocess – can be really associated with the velocity of *transmitted particles* (see Section III C).

**(III) This phenomenon is understood as a "pulse reshaping" process, in which a medium preferentially attenuates the later parts of an incident pulse, in such a way that the output peak appears shifted towards earlier times:**

Both the usual and generalized Hartman effects have been found for the *elastic* scattering process, i.e., for a passive medium in the layer (photon barrier). Thus, the phrase "a medium preferentially *attenuates*" is inappropriate for this case – the layer of a passive medium *separates* the to-be-transmitted part of the incident pulse from its to-be-reflected part, without any attenuation. A "pulse reshaping" process results from the interference between two coherently evolved subensembles – to-be-transmitted and to-be-reflected.

Thus, it is not surprising that "the output peak appears shifted towards earlier times" – any interference maximum in the barrier region, as being a result of a coherent superposition of two subprocesses, can move with any velocity because no energy or matter transfer is associated with its motion. The above shift does not at all mean that the genuine velocity of particles is superluminal in this case. This fact could be associated with tunneling if only reflection was absent. However, superliminal "tunneling velocities" appear just in the case of a *non*-resonant tunneling.

Whether or no the shape of the transmitted pulse differs from that of the incident pulse, this fact is of secondary importance because there is no causal relationship between these two pulses when reflection is nonzero. The above "pulse reshaping process" says nothing about the tunneling (sub)process (see also [16]). It also cannot serve as a physical mechanism of "the quantum speed up" of transmitted particles. The latter is merely the artefact of the concepts of the Wigner and Buttiker(-Larmor) tunneling times, which violate the (macro-)causality principle. In this connection, it is important to carefully analyze the next phrase in the above quotation.

**(IV) Although the apparent tunneling velocity… is superluminal, this is not a genuine signal velocity, and Einstein causality is not violated:**

By a genuine signal velocity is meant here (see [30]) the propagation velocity of the abrupt leading edge of the light pulse tunneling through the barrier region (see also [35–39]). But why this concept is treated here as more fundamental than that of the group velocity – by Büttiker and Landauer [23], not only the CM of the incident wave packet but also its "abrupt leading edge" (if any) do not transform into the corresponding points of the transmitted wave packet. However, as is said in [42] "Fronts are preserved in the output. Therefore, although there is no physical law which guarantees that an incoming peak turns into outgoing peak, there is a physical law namely causality, that guarantees that an incoming front turns into an outgoing front, even when the front carriers little energy or probability".

To justify this statement, Chiao and Steinberg [42] suggest the idealized model of a "black box" which locally relates an input to an output wave form by means of a linear transfer function. They show that the relationship between the input and output is causal in this model when the Fourier transform of this transfer function obeys the Kramers-Kronig relations. And they claim then, with referring to Jackson [44], that "the generalization of this argument to *propagation* through any *spatially extended* "black box" that is linear and causal, is straightforward".

However, the reference to Jackson [44] is inappropriate here, because no part in this textbook concerns the problem under consideration. At first glance, it is the exercise 7.8 on the page 234 that has a direct bearing on this problem. Indeed, this exercise is posed as follows:

> "A very long plane-wave train of frequency $\omega_0$ with a sharp front edge is incident normally from vacuum on a semi-infinite dielectric described by an index of refraction $n(\omega)$ and occupying the half-space $x > 0$. Just outside the dielectric (at $x = 0$) the *incident* electric field is $E_0(0,t) = \theta(t)e^{-\epsilon t}\sin(\omega_0 t)$, where $\theta(t)$ is the step function… The exponential decay constant $\epsilon$ is a positive infinitesimal…;
>
> (b) Prove that a sufficient condition for causality (that no signal propagate faster than the speed of light in vacuum) in this problem is that the index of refraction as a function of *complex* $\omega$ be an analytic function, regular in the upper half $\omega$ plane with nonvanishing imaginary part there, and approaching unity for $|\omega| \to \infty$.

(c) Generalize the argument of (b) to apply to any incident wave train."

However, as is seen, the boundary condition for $E_0(0,t)$ at the point $x = 0$ does not correspond to the phrase "a long plane-wave train... with a sharp front edge is *incident normally from vacuum on a semi-infinite dielectric...*" In reality, this problem deals with the wave field which is generated at the left boundary $x = 0$ of the semi-infinite dielectrics and propagates into the region $x > 0$ occupied by this dielectric. Unlike the OCS, in this exercise there is no reflection and, thus, there is no splitting of the incident wave packet into two coherently evolved parts. So that the problem considered in this exercise has nothing in common with that concerned in the statement (ɪv).

Note, Sokolovski [20] unlike Chiao and Steinberg refers to the chap. 3 of the book [45] for supporting the statement (ɪv). However, in our opinion this reference misleads too. The dispersion-relation argument is applied in [45] only to *one*-channel scattering processes, when the incoming pulse does not split within the "black box" into several outgoing channels.

In the case of a non-resonant tunneling, we deal with a two-channel scattering process, and the dispersion-relation argument is insufficient here for proving or disproving the transmission channel as being governed by the Einstein causality. As regards the "reshaping process", this "process" is evident to be noncausal, because it relates the incident pulse to the transmitted one which is causally disconnected with the former (see [23]). Thus, neither the statement (ɪv) by Chiao and Steinberg, nor the statement "Tunneling Confronts Special Relativity" by Nimtz can be considered as proven. Moreover, both are wrong: the former to concern the "reshaping process" is falsified in the paper [23] where it is pointed out that "there is a physical law namely causality, that guarantees that an incoming front" does not turn, in the case of the OCS, into the transmitted front. As regards the latter to concern exactly the tunneling subprocess, it is falsified in the alternative model of the OCS [1–4].

Perhaps, in the most precise manner, the internally conflicting character of the logics to underlie the "pulse-reshaping argument" is reflected in the following statement: "... the causality is not violated since reshaping destroys causal relationship between the incident and the transmitted peaks" [20].

Thus, the emergence of tunneling velocities greater than $c$, in the tunneling-time concepts elaborated on the basis of the contemporary model of the OCS, undermines the legitimacy of this model, rather than the legitimacy of the fundamental concepts of the wave dynamics – the group and (probability or energy) flow velocities. This is a signal to revise the contemporary model of the OCS, rather than to diminish (as it is done at present) these fundamental concepts in favour of the concept of the signal velocity associated with the "information transfer". The "energy transfer" and "matter transfer" must not violate the "naive", (macro-)causality principle. As regards the "information transfer", this notion can be filled with a physical content only in the context of the notions of the "energy transfer" and "matter transfer" (on some problems associated with this concept see, e.g., [31]).

### I. Superluminal tunneling velocities as "weak values"

Saying about the pulse-reshaping process, Chiao and Steinberg [42] offer one more argument for supporting the appearance of superluminal tunneling velocities. They write in [42] that such anomalous velocities are in a full agreement with the idea of a "weak measurement": "It has been shown by Aharonov and Vaidman... that when a "weak measurement" ... is made on a subensemble defined both by state preparation and by a postselection of low probability, mean values can be obtained which would be strictly forbidden for any complete ensemble." By the proponents of the "weak measurement" approach this agreement justifies the significance of the Hartman effect (see also [46–50]).

However, as was shown above, the timekeeping procedures to underlie the tunneling-time concepts and experiments violate the causality principle. Thus, the above agreement cannot improve the bad reputation of the widespread interpretation of the Hartman effect as that saying about superluminal tunneling velocities. This fact rather puts into question the very "weak-measurement" idea (its careful analysis can be found in the recent papers by Svensson [51, 52]). As it will be shown below, the tunneling time concepts introduced by Steinberg [47] on the basis of this idea violates the causality principle too.

As is known (see [50]), according to this idea the "weak value" of a variable $A$ for the preselected and postselected subensemble is

$$A_w = \frac{< \psi_f | A | \psi_{in} >}{< \psi_f | \psi_{in} >}; \tag{12}$$

where $|\psi_{in} >$ and $|\psi_f >$ are, respectively, the initial and final states of a selected subensemble of particles.

On the basis of this formula Steinberg introduces in [47] the conditional transmission dwell time for a particle with a given energy, scattering on the rectangular barrier (see also [46]). In doing so, he assumes that the initial state $|\psi_{in}>$ is linked to the final state $|\psi_f>$ (the transmitted wave) "…by a parity flip combined with a time reversal…"

However, Steinberg does not prove that "a parity flip combined with a time reversal" does yield the preselected initial state which is causally linked to the postselected transmitted state. Moreover, as is shown in Section III A, the initial state causally linked to this final state at the midpoint of the barrier region differs from that used by Steinberg. Thus, the time scale defined in [47] cannot be really treated as a "weak value", because the "measurement" that could be associated with this value "…is [in fact not] made on a subensemble defined *both* by state preparation *and* by a postselection…" That is, the timekeeping procedure presented in [47], like the standard concepts of the tunneling velocity, violates the causality principle – both in the Steinberg and Wigner timekeeping procedures the initial (preselected) and final (postselected) states describe the causally disconnected statistical ensembles. Hence it is not surprising that the timekeeping procedure [47] leads too to superluminal velocities.

### J. On the Winful reinterpretation of the existing tunneling-time experiments

One more widely-known attempt to reconcile the Hartman effect with the causality principle was undertaken by Winful [16]. Considering electromagnetic waves he noted that "Wave propagation in any medium (including vacuum) proceeds through the storage and release of energy". That is, Winful prefers to consider the transfer of the electromagnetic energy through a photonic barrier as its accumulation in the barrier region and the subsequent outflow from this region.

Of course, to divide the process of transferring the electromagnetic energy through the barrier region onto two stages – the energy accumulation and its subsequent release – is a matter of taste. But this step implies that the duration of the energy transfer must be defined then as the sum of the "accumulation time" and the "release time". At the same time Winful associates the duration of the energy transfer only with "…a lifetime of stored energy *leaking out* of both ends of the barrier…". He specifies that its duration "…is not the time it takes for the input peak to propagate to the exit since the pulse does not really propagate through the barrier…What is really measured is the lifetime of stored energy *escaping* through both ends" (italics supplied). That is, in fact, the accumulation stage turns out to be beyond the framework of Winful's timekeeping procedure – the process of transferring (tunneling) the electromagnetic energy through the barrier region is changed in this procedure by that of escaping the stored energy from this region. Thus, Winful's explanation of the Hartman effect is moot. It does not provide a way out of the deadlock situation in the TTL.

### K. On the Bohmian approach to the tunneling time problem

So, none of the existing approaches to the TTP allows a correct resolution of this problem. All they contain the same shortcoming – being unable to describe the tunneling subprocess at all stages of scattering, they resort to unproven assumptions about its dynamics, which really contradict the (macro-)causality principle.

At first glance, the Bohmian approach to the OCS (see, e.g., [27, 53, 54]) does not suffer from this shortcoming, because its "causal" trajectories defined for transmitted and reflected particles occupy the non-overlapping spatial regions at all stages of scattering. But this is not. The causality of these trajectories is illusive. The very fact that at the initial instant of time, that is, long before the scattering event, the sets of to-be-transmitted and to-be-reflected trajectories start from the different spatial regions (separated by the point whose location depends on the shape of the distant potential barrier) means that the Bohmian model of the OCS contradicts the causality principle.

However, this fact does not at all mean that the Bohmian approach is invalid by itself. As P. Holland said, "This suggests the problem is intrinsic to the quantum formalism and not an artefact of one particular [Bohmian] interpretation" (see p. 110 in [53]). Indeed, from the mathematical point of view, the Bohmian approach coincides with QM. The only difference between them is that the former interprets the lines of the probability flow as particle's trajectories, and nothing more. Thus, all defects of the contemporary quantum-mechanical description of the OCS turn into those of the Bohmian model of this process.

It is worthwhile to note that, in fact, the well-known wave-packet analysis and Bohmian approach are two sides of the same "coin" – quantum mechanics. The former visualizes the results of monitoring the quantum probability associated with a time-dependent wave function, and the latter visualizes those of monitoring the

corresponding probability flow. Thus, the above mystery with Bohmian trajectories relates, in fact, to the pathological properties of the very wave function $\Psi_{tot}(x,t)$ to describe the OCS. Indeed, the probability flow lines in the quantum-mechanical model of this process, behaving contrary to the causality principle, allow tracing the subprocess dynamics at all stages of scattering. But the probability density does not. (And the corresponding model in CED possesses the same feature.)

### L. Summing-up: The major defect of the conventional quantum-mechanical description of a 1D completed scattering

So, our analysis of the existing tunneling-time concepts shows that none of them, including the dwell, group and Larmor tunneling times, can serve as measure of the duration of a non-resonant tunneling. Within the contemporary model of the OCS, all three fundamental concepts can be correctly applied neither to the whole scattering process nor to its subprocesses, transmission and reflection.

The reason lies, eventually, in the contradictory properties of the wave function $\Psi_{tot}(x,t)$ that describes the OCS – the corresponding quantum probability density $|\Psi_{tot}(x,t)|^2$ (whose dynamics is usually considered within the wave-packet analysis) shows the separability of transmission and reflection only at the *final* stage of scattering; while the behavior of the lines of the corresponding probability flow density (which is usually considered within the Bochmian approach) shows their separability at *all* stages of scattering. This means that $|\Psi_{tot}(x,t)|^2$ cannot be unambiguously interpreted as the probability density to describe the whole ensemble of scattered particles, and the flow lines ending in the transmission (reflection) region cannot be correctly identified as the one to describe transmitted (reflected) particles.

The alternative approach [1–4, 55] solves this problem. By providing the formalism of a separate description of transmission and reflection at all stages of scattering, both in QM and in CED (see the model [56]), it gives the way to get rid of paradoxes which overcrowd the current TTL.

## III. A 1D COMPLETED SCATTERING AS A COMPLEX PROCESS TO CONSIST OF TWO COHERENT ALTERNATIVE SUBPROCESSES – TRANSMISSION AND REFLECTION

### A. Wave functions for transmission and reflection

The main idea of the alternative approach [1–4, 55] is to represent the OCS, for any semitransparent potential barrier $V(x)$, as a complex quantum process consisting of two inseparable alternative subprocesses – transmission and reflection. For any value of $k$ the total stationary wave function $\Psi_{tot}(x;k)$ (1) is implied can be uniquely presented as a coherent superposition of two subprocess wave functions to describe these subprocesses at all stages of scattering, each of these functions having one incoming and one outgoing waves 'causally' matched to each other at some spatial point.

Let $\psi_{tr}(x;k)$ and $\psi_{ref}(x;k)$ be the searched-for subprocess wave functions to describe transmission and reflection, respectively; in $\psi_{tr}(x;k)$ ($\psi_{ref}(x;k)$) the incident wave is matched to the transmitted (reflected) one at some joining point. Thus, the program of searching for these functions is reduced in fact to finding two incident waves that are matched causally to the corresponding (known) outgoing ones and, besides, obey the condition $\psi_{tr}(x;k) + \psi_{ref}(x;k) = \Psi_{tot}(x;k)$.

The peculiarity of this task is that, for a non-resonant tunneling through the semitransparent potential barrier, there is no solution to the stationary Schrödinger equation, which would be everywhere continuous together with its first derivative and have simultaneously one incoming and one outgoing waves. This means that the classes of wave functions to describe the OCS and its *sub*processes must differ from each other.

But this was to be expected. Indeed, if the subprocess wave functions could obey the continuity requirements inherent to the dynamics of usual quantum processes, the transmission and reflection could occur separately from each other. But this is impossible for a non-resonant tunneling. Thus, weakening the continuity requirements at the joining point is necessary for the subprocess wave functions. Of importance is that they must ensure the causal relationship between the ensembles described by the outgoing and incoming waves. We postulate that this takes place when the subprocess wave functions as well as the corresponding probability flow densities are continuous at the joining point.

As was shown in [2, 4, 55], for symmetrical barriers this point coincides for any value of $k$ with the midpoint of the barrier region, and the wave functions for reflection and transmission to obey these continuity

requirements read as follows. For $x \leq a$

$$\psi_{ref}(x;k) = A_{ref}^{in} e^{ikx} + b_{out} e^{ik(2a-x)}, \qquad \psi_{tr}(x;k) = A_{tr}^{in} e^{ikx}; \tag{13}$$

for $a \leq x \leq x_c$

$$\begin{aligned} \psi_{ref}(x;k) &= \kappa^{-1}\left(PA_{ref}^{in} + P^* b_{out}\right) e^{ika} F(x - x_c; k) \\ \psi_{tr}(x;k) &= \kappa^{-1} P A_{tr}^{in} e^{ika} F(x - x_c; k) + b_{full} G(x - x_c; k); \end{aligned} \tag{14}$$

for $x \geq x_c$

$$\psi_{ref}(x;k) \equiv 0, \qquad \psi_{tr}(x;k) \equiv \Psi_{full}(x;k); \tag{15}$$

$$A_{ref}^{in} = b_{out}^* \left(b_{out} + a_{out}\right) \equiv \sqrt{R}(\sqrt{R} \pm i\sqrt{T}), \quad A_{tr}^{in} = a_{out}\left(a_{out}^* - b_{out}^*\right) \equiv \sqrt{T}(\sqrt{T} \mp i\sqrt{R}); \tag{16}$$

for any value of $k$ either the upper or lower signs are valid.

Simple analysis shows that for symmetric potential barriers $|\psi_{tr}(x_c - x;k)| = |\psi_{tr}(x - x_c;k)|$. Besides, letting $\psi_{tr}(x;k) = M_{tr}(x;k)\exp[i\phi_{tr}(x;k)]$, we obtain that at the joining point $x_c$

$$\phi_{tr}(x_c - 0;k) = \phi_{tr}(x_c + 0;k), \qquad \left.\frac{\partial \phi_{tr}(x;k)}{\partial x}\right|_{x=x_c-0} = \left.\frac{\partial \phi_{tr}(x;k)}{\partial x}\right|_{x=x_c+0}, \tag{17}$$

$$M_{tr}(x_c - 0;k) = M_{tr}(x_c + 0;k), \qquad \left.\frac{\partial M_{tr}(x;k)}{\partial x}\right|_{x=x_c-0} = -\left.\frac{\partial M_{tr}(x;k)}{\partial x}\right|_{x=x_c+0} \neq 0.$$

That is, for symmetrical potential barriers the values of the subprocess wave function $\psi_{tr}(x;k)$ and its first derivative $\partial\psi_{tr}(x;k)/\partial x$ in the limit $x \to x_c - 0$ can be calculated through those of the total wave function $\Psi_{tot}(x;k)$ and its first derivative at the point $x_c$.

So, by this approach, reflected particles never cross the point $x_c$. This result agrees with the well known fact that, for a classical particle impinging from the left a smooth symmetrical potential barrier, the midpoint of the barrier region is the extreme right turning point, irrespective of the particle mass as well as the symmetrical form of the barrier and its size. Thus, in the regions $x < x_c$ and $x > x_c$ particles move under different physical contexts (see [55] and references therein) – to the right of the midpoint $x_c$ the field of any symmetric potential barrier is such that it cannot turn a particle, having crossed the midpoint $x_c$ from the left, back into the region $x < x_c$. As a result, the reflection subprocess does not run to the right of this point: $\psi_{ref}(x;k) \equiv 0$ in the region $x \geq x_c$. As regards the transmission subprocess, to the left of this point it runs on the background of the reflection one – $\psi_{tr}(x;k) = \Psi_{tot}(x;k) - \psi_{ref}(x;k)$; to the right of the point $x_c$ this background disappears – $\psi_{tr}(x;k) = \Psi_{tot}(x;k)$. So, the other physical context, the other solution of the Schrödinger equation describes the subprocess dynamics. And what is also important is that the transfer from the one context to the other one occurs, in the stationary case, without breaking the unitary evolution of the transmission subprocess.

The stationary wave functions $\psi_{tr}(x;k)$ and $\psi_{ref}(x;k)$ found for any value of $k$ lead to the unique decomposition of the time-dependent wave function (2) with any given function $\mathcal{A}(k)$: $\Psi_{tot}(x,t) = \psi_{tr}(x,t) + \psi_{ref}(x,t)$ where

$$\psi_{tr,ref}(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{A}(k)\psi_{tr,ref}(x;k); e^{-iE(k)t/\hbar} dk;$$

the time-dependent wave functions $\psi_{tr}(x,t)$ and $\psi_{ref}(x,t)$ describe, respectively, the transmission and reflection subprocesses at all stages of scattering. Their norms are

$$\langle \psi_{tr}(x,t)|\psi_{tr}(x,t)\rangle = \mathbf{T}, \qquad \langle \psi_{ref}(x,t)|\psi_{ref}(x,t)\rangle = \mathbf{R};$$

where **T** and **R** are, respectively, the transmission and reflection coefficients.

In the limit $l_0 \to 0$, i.e., when $\mathcal{A}(k) \to \delta(k - k_0)$, these norms are constant in time and obey the equality

$$\mathbf{T} + \mathbf{R} = \langle \Psi_{tot}(x,t)|\Psi_{tot}(x,t)\rangle = 1 \tag{18}$$

to be characteristic for mutually exclusive random events; in this limit $\mathbf{T} = T(k_0)$, $\mathbf{R} = R(k_0)$.

Thus, in this limit, transmission and reflection are alternative subprocesses in spite of the existence of interference between them. However, this limiting case is inappropriate for revealing the inherent properties of the time-dependent subprocess dynamics. Any real wave packet, even with a however large but finite value of the parameter $l_0$, possesses the front and tail parts where the role of all harmonics (not only $k_0$) is essential. This fact is important for understanding the time-dependent dynamics of the transmission subprocess.

As it follows from our analysis, for finite values of $l_0$ Eq. (18) is true for the initial and final stages of scattering, i.e., long before and long after the scattering event. For the final stage of the OCS this statement is obvious since the transmitted and reflected wave packets are localized at this stage in the disjoint spatial regions. As regards the initial stage, the incident wave packets $\psi_{tr}^{in}(x,t)$ and $\psi_{ref}^{in}(x,t)$ of the subprocesses interfere with each other. However, as is seen from (16), for each harmonic $k$, not only $A_{tr}^{in}(k)+A_{ref}^{in}(k)=1$ but also $|A_{tr}^{in}(k)|^2+|A_{ref}^{in}(k)|^2=1$. As a consequence,

$$\langle\psi_{tr}^{in}(x,t)|\psi_{ref}^{in}(x,t)\rangle=\pm i\int_{-\infty}^{\infty}|\mathcal{A}(k)|^2\sqrt{T(k)R(k)}dk.$$

That is, $\Re\langle\psi_{tr}^{in}(x,t)|\psi_{ref}^{in}(x,t)\rangle=0$ and hence the equality (18) is satisfied. This proves our assertion.

Note, at the very stage of scattering the norm $\mathbf{T}$ varies in the general case because now $d\mathbf{T}/dt=I_{tr}(x_c+0,t)-I_{tr}(x_c-0,t)\neq 0$; here $I_{tr}$ is the probability current density to correspond to the wave function $\psi_{tr}(x,t)$. In the stationary case, when only the main harmonic $k_0$ exists, this difference is zero. However, when we deal with the localized wave packet $\psi_{tr}(x,t)$ the input and output flows – $I_{tr}(x_c-0,t)$ and $I_{tr}(x_c+0,t)$ – do not balance each other. This effect is maximal when the front or tail part of the wave packet crosses the midpoint $x_c$. In these two cases, subharmonics to enter into $\psi_{tr}(x,t)$ play an essential role; the terms in $I_{tr}(x_c+0,t)-I_{tr}(x_c-0,t)$ that describe the interaction of the main harmonic $k_0$ with subharmonics lead to nonzero values of this quantity due to the weakened continuity requirements imposed at the point $x_c$ on the subprocess wave function $\psi_{tr}(x;k)$.

At the same time the total variation of $\mathbf{T}$ is zero:

$$\langle\psi_{tr}^{in}(x,t)|\psi_{tr}^{in}(x,t)\rangle=\langle\Psi_{tr}(x,t)|\Psi_{tr}(x,t)\rangle.$$

Of importance is also to stress that $\mathbf{R}$ is constant even at the stage of scattering: $I_{ref}(x_c+0,t)=I_{ref}(x_c-0,t)=0$ for any value of $t$ because $\psi_{ref}(x_c,t)=0$. All this means that in the time-dependent case the reflection dynamics is unitary, as in the stationary case. As regards the time-dependent transmission dynamics, figuratively speaking, it is 'asymptotically' unitary.

So, the transmission and reflection subprocesses can be treated as strictly alternative ones only in the stationary case. In the papers [1–4, 55] this fact is treated as that to undermine the idea of decomposing the OCS into subprocesses in the time-dependent case. But now we assess this fact otherwise.

Firstly, the above (univariant) decomposition of the total wave function $\Psi_{tot}(x,t)$ into the subprocess ones $\psi_{tr}(x,t)$ and $\psi_{ref}(x,t)$ realizes the only possibility to reveal, within the standard wave-packet analysis, the individual dynamics of the transmission and reflection subprocess at all stages of scattering. Secondly, as was shown above, the subprocess dynamics cannot be described by wave functions which would be everywhere continuous together with their first derivatives; otherwise the transmission and reflection subprocesses could be considered as usual (unitary) quantum processes taking place singly when $\mathbf{R}\neq 0,1$ – the fact that the joining point $x_c$ serves as a 'sink' or 'source' for the to-be-transmitted subensemble of particles should be treated as an inherent property of the time-dependent transmission subprocess.

So, the alternative model creates a cardinally new situation in solving the TTP since the quantum dynamics of tunneling (transmission) is now known at all stages of scattering (of importance is also to stress that this approach is fully applicable to the tunneling phenomenon in CED (see [56])). Our next step is to define the dwell and group characteristic times for transmission and reflection, as well as to consider the possibility of their measurement within the Larmor-clock timekeeping procedure. In doing so, we will pay a particular attention to the case of tunneling a particle through the rectangular barrier in the opaque limit $d\to\infty$, when the group tunneling velocity must be superluminal by the standard approaches.

### B. Dwell times for transmission and reflection

Let us again, as in Section II C, apply the flow-velocity concept and the corresponding dwell-time concept for introducing the tunneling (or, more generally, transmission) velocity and time in the stationary case. The

transmission dwell time $\tau_{dwell}^{tr}$ reads as

$$\tau_{dwell}^{tr}(k) = \frac{1}{I_{tr}} \int_a^b |\psi_{tr}(x;k)|^2 dx. \tag{19}$$

As is seen, this time scale unlike the dwell times (5) and (6) is unambiguously associated with the transmission subprocess. We consider the quantity $v_{flow}^{tr}(x) = I_{tr}/|\psi_{tr}(x;k)|^2$ as the one giving the genuine average transmission velocity of particles. While the "velocities" $v_{flow}^{(1)}$ and $v_{flow}^{(2)}$ in Section II C have no physical meaning.

The reflection dwell time $\tau_{dwell}^{ref}$ is introduced in a similar way –

$$\tau_{dwell}^{ref}(k) = \frac{1}{I_{ref}} \int_a^{x_c} |\psi_{ref}(x,k)|^2 dx, \qquad I_{ref} = \frac{\hbar k}{m} R(k).$$

It should be stressed however that $\tau_{dwell}^{ref}(k)$ depends not only on the average velocity of reflected particles, but also on the average depth of their penetration into the barrier region. Therefore the expression $I_{ref}/|\psi_{ref}(x;k)|^2$ cannot be interpreted as the average velocity of reflected particles at the point $x$.

It is useful here to compare the transmission dwell time $\tau_{dwell}^{tr}$ for a particle tunneling through the rectangular barrier of height $V_0$ ($E < V_0$) (see [1, 3]) with the Buttiker dwell time $\tau_{dwell}^{(2)}$ [15]:

$$\tau_{dwell}^{tr} = \frac{m}{2\hbar k \kappa^3} \left[ \left( \kappa^2 - k^2 \right) \kappa d + \kappa_0^2 \sinh(\kappa d) \right], \quad \tau_{dwell}^{(2)} = \frac{mk}{\hbar \kappa} \cdot \frac{2\kappa d(\kappa^2 - k^2) + \kappa_0^2 \sinh(2\kappa d)}{4k^2\kappa^2 + \kappa_0^4 \sinh^2(\kappa d)}; \tag{20}$$

$\kappa_0 = \sqrt{2mV_0}/\hbar$.

As is seen, unlike $\tau_{dwell}^{(2)}$ the transmission dwell time $\tau_{dwell}^{tr}$ increases exponentially in the limit $d \to \infty$, rather than saturates. Thus, in our approach the dwell transmission time does not lead to the Hartman effect. By this approach the opaque barrier strongly retards on average the motion of the to-be-transmitted subensemble of particle with a given energy $E$, when it enters the barrier region. (Note that the dwell time $\tau_{dwell}^{(1)}$ (see (5)) does not saturate in this limit too. However, as was shown in Section II C, the physical meaning of this quantity is unclear.)

Moreover, as is shown in [5] (see also [55]), for a particle tunnelling through the system of two identical rectangular barriers of width $d$ and height $V_0$, with the distance $l$ between them, our approach does not lead to the generalized Hartman effect predicted in [18]. Indeed, as the transmission dwell time $\tau_{dwell}^{tr}$ possesses the property of additivity, it can be written for this case in the form

$$\tau_{dwell}^{tr} = \tau_{tr}^{(1)} + \tau_{tr}^{gap} + \tau_{tr}^{(2)};$$

$\tau_{tr}^{(1)}$ describes the first barrier; $\tau_{tr}^{gap}$ does the gap between the barriers, and $\tau_{tr}^{(2)}$ describes the second barrier. In the opaque limit, when $V_0 \to \infty$ (or $\kappa_0 \to \infty$) and $d$ is fixed, we have (see [5, 55])

$$\tau_{tr}^{(1)} = \tau_{tr}^{(2)} \approx \frac{m}{4\hbar k \kappa_0} \exp(2\kappa_0 d), \quad \tau_{tr}^{gap} \approx \frac{m\kappa_0^2}{8\hbar k^4} \left[ kl - \sin(kl) \right] \exp(2\kappa_0 d).$$

As is seen, $\tau_{dwell}^{tr}$ increases exponentially in this case, and what is also important is that it depends on the distance $l$ between the barriers. Moreover, a tunneling particle spends the most time just in the space between the opaque barriers. As regards the Buttiker dwell time and Wigner phase time, in this limit they do not depend on the distance between the barriers and tend to zero when $\kappa_0 \to \infty$.

Analogous situation arises in another opaque limit, when $d \to \infty$ and $V_0$ is fixed. Explicit expressions for the dwell times becomes somewhat complicated (see [5, 55]) in this case, and, as not giving a qualitatively new information, they are omitted here. The only difference is that $\tau_{tr}^{(1)}$ and $\tau_{tr}^{(2)}$ saturate in this case, rather than tend to zero.

For the following it is also important to stress that the transmission dwell time possesses the property of additivity. This means that, if $\tau_{dwell}^{tr}(k)$ is the transmission dwell time for the barrier region, then the one for the asymptotic spatial region $[0, b+L]$ (with $L = a$, for example) will be $\tau_{dwell}^{tr}(k) + 2mL/\hbar k$. That is, the dwell transmission time for the asymptotical spatial region to include the rectangular barrier increases, too, exponentially in the opaque limit $d \to \infty$.

### C. Asymptotic group times for transmission and reflection

Let us now consider the transmission dynamics in the interval $[0, b + L]$ from the point of view of the group-velocity concept. Let $X_{tr}(t)$ and $X_{ref}(t)$ be, respectively, the positions of the CMs of the wave packets $\psi_{tr}(x,t)$ and $\psi_{ref}(x,t)$ at the instant of time $t$:

$$X_{tr}(t) = \frac{1}{\mathbf{T}} < \psi_{tr}(x,t)|\hat{x}|\psi_{tr}(x,t) >, \qquad X_{ref}(t) = \frac{1}{\mathbf{R}} < \psi_{ref}(x,t)|\hat{x}|\psi_{ref}(x,t) > .$$

Fig. 1 obtained for the 'opaque' rectangular barrier shows the results of tracing the CM's position of the wave packet $\psi_{tr}(x,t)$; $a = 200nm$, $b = 215nm$, $V_0 = 0.2eV$, $(\hbar k_0)^2/2m = 0.05eV$ and $l_0 = 10nm$.

As is seen, in the asymptotically remote regions the CM's velocity coincides with $\hbar k_0/m$. However, the CM's velocity in the barrier region (see the almost flat part of the curve) is much smaller than the one in the asymptotically remote regions where the wave packet moves freely. That is, the group-velocity concept justifies the effect of retardation of the to-be-transmitted wave packet in the barrier region, which was predicted by the flow-velocity concept in the opaque limit. Note that in the case considered the wave packet $\psi_{tr}(x,t)$ is wider than the barrier. So that when the CM of this packet moves in the barrier region its front and tail parts are located beyond this region. In this case the main harmonics $k_0$ dominates in the barrier region and, thus, its interaction at the midpoint $x_c$ with subharmonics is negligible; as a consequence, at this stage this point does not influence the norm $\mathbf{T}$ and the CM's position (see also Section III A).

However, its influence is essential, even for this narrow in $k$ space wave packet, when the wave-packet's front and tail parts cross this point. Namely, when its *front* part crosses the midpoint of the region of the opaque rectangular barrier this point serves as a source of particles. This results in accelerating the CM of the wave packet, without accelerating the particles of the corresponding subensemble. In contrast, when the tail part of the packet crosses this point, then it serves as a 'sink' for this subensemble. And, since the main body of the packet is now to the right of the point, the disappearance of particles in the point $x_c$ leads again to accelerating the CM of the packet (and again, without accelerating the particles). As a result, the group transmission time to describe the CM's dynamics in the interval $[0, b + L]$ proves to be "anomalously" short.

The transmission and reflection group times for this asymptotically large interval can be defined as follows. Let $t_{depart}^{tr}$ and $t_{arrive}^{tr}$ be such instants of time that

$$X_{tr}(t_{depart}^{tr}) = 0; \qquad X_{tr}(t_{arrive}^{tr}) = b + L, \tag{21}$$

Then the transmission time $\Delta t_{tr}$ for this interval can be defined as the difference $t_{arrive}^{tr} - t_{depart}^{tr}$.

Similarly, let $t_{depart}^{ref}$ and $t_{arrive}^{ref}$ be such instants of time that

$$X_{ref}(t_{depart}^{ref}) = X_{ref}(t_{arrive}^{ref}) = 0; \tag{22}$$

$t_{depart}^{ref}$ and $t_{arrive}^{ref}$ are, respectively, the smallest and largest roots of Eq. (22); note that since $a \gg l_0$ these roots are evident to exist. Then the reflection time $\Delta t_{ref}$ can be defined as follows: $\Delta t_{ref} = t_{arrive}^{ref} - t_{depart}^{ref}$.

Note, since all quantities in (21) and (22) are associated with the asymptotically remote spatial regions, we can calculate them for the incoming and outgoing wave packets (3). In doing so, it is suitable to rewrite the amplitudes $A_{ref}^{in}$ and $A_{tr}^{in}$ (see (16)) in the form,

$$A_{ref}^{in} = 1 - A_{tr}^{in} \equiv \sqrt{R(k)}e^{i\lambda(k)}, \qquad \lambda(k) = \arg\left[b_{out}^{*}\left(b_{out} + a_{out}\right)\right]. \tag{23}$$

Then, for the limiting case $l_0 \to \infty$, the CM positions $X_{tr}(t)$ and $X_{ref}(t)$ at the initial and final stages of scattering are defined as follows (for comparison, the CM position $X_{tot}(t)$ for the total wave packet at the initial stage of scattering is presented here too),

(a) long before the scattering event

$$X_{tr}(t) = X_{ref}(t) = X_{subpr}^{in}(t) \equiv \frac{\hbar k}{m}t - \lambda'(k), \qquad X_{tot}^{in}(t) = \frac{\hbar k}{m}t;$$

(b) long after the scattering event

$$X_{tr}(t) = X_{tr}^{out} \equiv \frac{\hbar k}{m}t - J'(k) + d, \qquad X_{ref}(t) = X_{ref}^{out} \equiv -\frac{\hbar k}{m}t - J'(k) + 2a;$$

here the prime denotes the derivative on $k$.

Thus, taking into account these expressions in (21) and (22) we obtain

$$\Delta t_{tr} = \frac{m}{\hbar k}\left[J'(k) - \lambda'(k) + a + L\right], \quad \Delta t_{tr} = \Delta t_{ref}(L) = \frac{m}{\hbar k}\left[J'(k) - \lambda'(k) + 2a\right];$$

here $k$ substitutes for $k_0$.

And, lastly, excluding from these expressions the terms to describe the outer spatial regions, we obtain the asymptotic scattering times $\tau_{tr}^{as}$ and $\tau_{ref}^{as}$ for transmission and reflection, respectively:

$$\tau_{tr}^{as} = \tau_{ref}^{as} = \frac{m}{\hbar k}\left[J'(k) - \lambda'(k)\right]. \tag{24}$$

The corresponding delay times $\tau_{tr}^{del} = \tau_{tr}^{as} - md/\hbar k$ and $\tau_{ref}^{del} = \tau_{ref}^{as} - md/\hbar k$ read as

$$\tau_{tr}^{del} = \tau_{ref}^{del} = \frac{m}{\hbar k}\left[J'(k) - d - \lambda'(k)\right]. \tag{25}$$

As is seen, unlike Exs. (7) for the corresponding time scales in the conventional model of the OCS, Exps. (24) and (25) contain the extra term $\lambda'(k)$.

For tunneling trough the rectangular barrier ($E < V_0$) the asymptotic transmission time $\tau_{tr}^{as}$ and the starting position $X_{tr}(0) \equiv X_{subpr}^{in}(0)$ are defined by the expressions (see [3])

$$\tau_{tr}^{as} = \frac{4m}{\hbar k \kappa}\,\frac{[k^2 + \kappa_0^2 \sinh^2(\kappa d/2)][\kappa_0^2 \sinh(\kappa d) - k^2 \kappa d]}{4k^2\kappa^2 + \kappa_0^4 \sinh^2(\kappa d)},$$
$$X_{subpr}^{in}(0) = -2\frac{\kappa_0^2}{\kappa}\,\frac{(\kappa^2 - k^2)\sinh(\kappa d) + k^2 \kappa d \cosh(\kappa d)}{4k^2\kappa^2 + \kappa_0^4 \sinh^2(\kappa d)}. \tag{26}$$

In the opaque limit $d \to \infty$, from Exps. (26) it follows that $\tau_{tr}^{as} \to \frac{2m}{\hbar k \kappa}$ and $X_{subpr}^{in}(0) \to X_{tot}^{in}(0) = 0$. These two results agree with the Fig. 1 that shows that, for tunneling through the opaque barrier, the curve $X_{tr}(t)$ indeed evolves from the origin and the asymptotic group transmission time $\tau_{tr}^{as}$ are much smaller than the dwell transmission time.

So, by this approach, in the limit $d \to \infty$ the flow-velocity and group-velocity concepts lead to cardinally different transmission times for the asymptotically large interval $[0, b + L]$ (in the case considered $\tau_{tr}^{as} \approx 0.024ps$, $\tau_{dwell}^{tr} \approx 0.155ps$). However, this fact does not mean that this approach is internally inconsistent. To prove this we have to address the Larmor-clock procedure [15] adapted in [3, 57] to the transmission and reflection subprocesses.

### D. Larmor times for transmission and reflection

In the considered setting of the problem for the rectangular barrier, the wave functions $\psi^{(\uparrow)}(x;k)$ and $\psi^{(\downarrow)}(x;k)$ to describe, respectively, the stationary states of particles with the spins parallel and antiparallel to the external infinitesimal magnetic field can be written as follows,

$$\psi_{tr,ref}^{(\uparrow,\downarrow)}(x;k) = \psi_{tr,ref}(x;k) \mp \omega_L \tilde{\psi}_{tr,ref}(x;k), \qquad \tilde{\psi}_{tr,ref}(x;k) = \frac{m}{2\hbar\kappa}\frac{\partial \psi_{tr,ref}(x;k)}{\partial \kappa}; \tag{27}$$

here $\omega_L = 2\mu B/\hbar$, $\mu$ and $B$ are, respectively, the absolute values of the electron magnetic moment and applied magnetic field. As was shown in [3, 57], in the limiting case $l_0 \to \infty$ the rotation angles $\Delta\phi_\perp^{tr,ref}$ and $\Delta\phi_\parallel^{tr,ref}$ to describe, respectively, the Larmor precession of the average spins of transmitted and reflected particles in the plane orthogonal to the magnetic field as well as their "aligning" with the field read as

$$\Delta\phi_\perp^{tr} \equiv -\omega_L(\tau_\perp^{tr} - \tau_{0\perp}^{tr}) = -\omega_L(\tau_{dwell}^{tr} + \tau_{flip}), \quad \Delta\phi_\parallel^{tr} \equiv -\omega_L(\tau_\parallel^{tr} - \tau_{0\parallel}^{tr}) = 0,$$
$$\Delta\phi_\perp^{ref} \equiv -\omega_L(\tau_\perp^{ref} - \tau_{0\perp}^{ref}) = -\omega_L \tau_{dwell}^{tr}, \quad \Delta\phi_\parallel^{ref} \equiv -\omega_L(\tau_\parallel^{ref} - \tau_{0\parallel}^{ref}) = 0; \tag{28}$$

where $\tau_{\perp,\parallel}^{tr,ref}$ and $\tau_{0\perp,\parallel}^{tr,ref}$ are the final and initial readings of the Larmor clocks to obey the relationships

$$\tau_\perp^{tr} = \tau_\perp^{ref} \equiv \tau_\perp, \quad \tau_{0\perp}^{tr} = \tau_{0\perp}^{ref} \equiv \tau_{0\perp} \neq 0, \quad T\tau_{0\parallel}^{tr} + R\tau_{0\parallel}^{ref} = 0; \quad \tau_{0,\perp}^{tr} = \tau_{0\parallel}^{tr}\sqrt{T/R} \quad (E < V_0); \tag{29}$$

$\tau^{tr}_{dwell}$ and $\tau^{ref}_{dwell}$ define the duration of the Larmor precession; $\tau_{flip}$ is defined by the expression

$$\tau_{flip}(k) = \frac{1}{kT(k)}\Re\Bigg[\psi_{tr}(x_c;k)\left(\frac{\partial\tilde{\psi}^*_{tr}(x_c+0;k)}{\partial x} - \frac{\partial\tilde{\psi}^*_{tr}(x_c-0;k)}{\partial x}\right) - \tilde{\psi}^*_{tr}(x_c;k)\left(\frac{\partial\psi_{tr}(x_c+0;k)}{\partial x} - \frac{\partial\psi_{tr}(x_c-0;k)}{\partial x}\right)\Bigg]. \tag{30}$$

With taking into account the relationships (17) and (27), this quantity can be rewritten via the total wave function $\Psi_{tot}$:

$$\tau_{flip}(k) = \frac{m}{\hbar k\kappa T(k)}\left(M_{tot}\frac{\partial^2 M_{tot}}{\partial x\partial\kappa} - \frac{\partial M_{tot}}{\partial x}\frac{\partial M_{tot}}{\partial\kappa}\right)\Bigg|_{x=x_c}; \tag{31}$$

here $M_{tot} = |\Psi_{tot}(x;k)|$.

Note, by the standard Larmor-clock procedure [15], for the problem considered (see Section II G)

$$\tau^{tr}_{0\perp} = \tau^{ref}_{0\perp} = \tau^{tr}_{0\|} = \tau^{ref}_{0\|} = 0, \quad \Delta\phi^{tr}_{\|} = -\omega_L\tau^{tr}_{\|} \neq 0, \quad \Delta\phi^{ref}_{\|} = -\omega_L\tau^{ref}_{\|} \neq 0.$$

Nonzero values of $\Delta\phi^{tr}_{\|}$ and $\Delta\phi^{ref}_{\|}$ say that this approach allows aligning the particle spin with the magnetic field, what contradicts QM. In our approach this effect is absent: $\tau^{tr}_{\|} = \tau^{tr}_{0\|}$ and $\tau^{ref}_{\|} = \tau^{ref}_{0\|}$.

Another important difference of the present Larmor-clock procedure from the standard one is that now there are *two* physical effects to influence the average spin of transmitted particles in the plane orthogonal to the magnetic field, rather than one. Apart from the already known Larmor precession of this spin under the magnetic field, whose duration is described by the dwell time $\tau^{tr}_{dwell}$, there appears a new effect – flipping the orthogonal projection of the average spin at the joining point $x_c$ – which is described by the quantity $\tau_{flip}$. This effect does not allow a direct measurement of $\tau^{tr}_{dwell}$. However, for reflected particles it does not appear because $\psi_{ref}(x_c,t) = 0$ (see (15)); the substitution $\psi_{ref}$ for $\psi_{tr}$ in (30) yields zero value. As a consequence, our approach eventually allows us to write $\tau^{ref}_{dwell}$ in terms of the physical quantities which can be directly measured by means of the Larmor-clock procedure.

As it follows from Eqs. (27) and (28), for both subprocesses in the case $E < V_0$ we have

$$\tau^{tr}_{dwell} + \tau_{flip} = \tau^{ref}_{dwell} = \tau_\perp - \tau_{0\perp} = \tau_\perp + \tau^{ref}_{\|}\sqrt{R/T} = \tau_\perp - \tau^{tr}_{\|}\sqrt{T/R} \tag{32}$$

where $T = 4k^2\kappa^2/[1+\kappa_0^4\sinh^2(\kappa d)]$; note, for $E > V_0$ the relationship between $\tau^{tr}_{0,\perp}$ and $\tau^{tr}_{0\|}$ (see (29)) is slightly different. As was shown in [3, 57], for $E < V_0$

$$\tau_\perp = \frac{mk}{\hbar\kappa}\ \frac{2(\kappa^2-k^2)\kappa d + \kappa_0^2\sinh(2\kappa d)}{4k^2\kappa^2 + \kappa_0^4\sinh^2(\kappa d)}, \quad \tau_{0\perp} = \frac{2mk}{\hbar\kappa}\ \frac{(\kappa^2-k^2)\sinh(\kappa d) + \kappa_0^2\kappa d\cosh(\kappa d)}{4k^2\kappa^2 + \kappa_0^4\sinh^2(\kappa d)}.$$

Thus, in the opaque limit $d \to \infty$, when the transmission dwell time $\tau^{tr}_{dwell}$ increases exponentially, the final readings $\tau_\perp$ of the Larmor clock saturate together with the reflection dwell time $\tau^{ref}_{dwell}$. In this case the time $\tau_{flip}$ is negative and its absolute value increases exponentially. The quantity $\tau_{0\perp}$ to describe the initial readings of the Larmor clock for both subprocesses tends to zero in this case; its analog in the timekeeping procedure based on the group-velocity concept, $\tau^{(0)}_{group} = -mX^{in}_{subpr}(0)/\hbar k$ (see (26), diminishes in this limit too.

## IV. ON THE HARTMAN EFFECT AND "SUPERLUMINAL TUNNELING VELOCITIES"

So, according to the timekeeping procedure based on the group-velocity concept, in the limits $d \to \infty$ and $l_0 \to \infty$, the asymptotic transmission time $\tau^{as}_{tr}$ saturates like the Wigner time $\tau_{ph}$ and the initial position $X^{in}_{subpr}(0)$ of the CM of the to-be-transmitted wave packet approaches $X^{in}_{tot}(0)$ (of course, $\psi_{tr}(x,t)$ does not approach $\Psi_{tot}(x,t)$ in this case). In fact the same result follows from the Larmor timekeeping procedure. Indeed, it says that like the dwell time $\tau^{(2)}_{dwell}$ (see (6)) the difference $\tau_\perp - \tau_{0\perp}$ between the final and initial readings of the Larmor clock saturates in this case with diminishing the initial reading $\tau_{0\perp}$.

At first glance, all this means that in this particular (but very important) case, our versions of these two timekeeping procedures coincide with those developed within the standard approaches, thereby justifying the existing claims about direct measurements of superluminal group tunneling velocities. However, this is not.

By our approach, only the flow-velocity (or, energy-velocity, in CED) concept allows revealing the genuine tunneling (transmission) velocity of particles and corresponding tunneling time – the time spent on average by transmitted particles in the barrier region. The main merit of this concept in studying the temporal aspects of tunneling, as a *sub*processes of the OCS, is that it does not characterize the transmitted wave packet as a whole. As a result, the dwell time introduced on its basis possesses the additivity property and does not "feel" the point $x_c$ together with the nonlinear effects to take place at this point.

As regards the timekeeping procedures based on tracing the CM of the subensemble of transmitted particles or tracing their average spin, both the wave-packet's CM and the average spin characterize the wave packet $\psi_{tr}(x,t)$ as a whole object. As a result, the role of those stages of scattering when the front and tail parts of the packet cross the point $x_c$ is essential even for wave packets with however large values of $l_0 \to \infty$. At these stages the "number of particles" in the transmitted subensemble is changed at the point $x_c$. As was shown in Section III C, in the opaque limit, this effect leads, at these stages, to the speed-up of the $\psi_{tr}(x,t)$'s CM (see Fig. 1) and eventually to the unexpectedly short asymptotic tunneling time $\tau_{tr}^{as}$. However, this effect has nothing to do with the tunneling velocity of particles!

Besides, this effect results in "flipping" the average spin of transmitted particles at the point $x_c$. That is, the value of $\tau_{flip}$ to describe the spin-flipping effect accrues just at the stages when the front and tail parts of the transmitted wave packet cross the point $x_c$. In the case of the opaque rectangular barrier, this effect leads to the situation when the final readings of the Larmor clock show the time to be much smaller than the average time spent by transmitted particles in the barrier region.

Thus, by our approach the observed superluminal tunneling velocity of the CM of the transmitted wave packet has nothing to do with the velocity of transmitted particles – the group-velocity concept is inapplicable for revealing the genuine (flow) velocity of particles taking part in the transmission subprocess. The same concerns the Larmor timekeeping procedure – because of the spin-flipping effect the final reading of the Larmor clock placed in the transmission zone does not give the genuine (dwell) time spent by transmitted particles in the barrier region.

Of importance is to stress that all the above concerns only the *transmission* time-dependent subprocess whose dynamics is non-unitary at the very stage of scattering. As regards the dynamics of the reflection subprocess, it is unitary at all stages of scattering. The group reflection time as well as the final readings of the Larmor clock located in the reflection zone are not affected by changing the physical context at the point $x_c$.

Moreover, taking into account the fact that the quantity $\tau_{flip}(x;k)$ is expressed through the total wave function $\Psi_{tot}(x;k)$ (see Exp. (31)), we arrive at conclusion that the Larmor timekeeping procedure allows testing our results obtained for both subprocesses. For this purpose one can use the relationships (28), (29) and (32). In particular, by our approach reflected particles do not cross the midpoint of any symmetric potential barrier. In this connection, it would be interesting to check the result that the quantity $\tau_{\parallel}^{ref}$ to coincide with $\tau_{0\parallel}^{ref}$ is indeed independent on the infinitesimal magnetic field switched on in any spatial interval located to the right of the point $x_c$.

## V. CONCLUSIONS AND DISCUSSION

So, our analysis of the tunneling-time concepts and experiments presented in the current TTL shows that all they are based on some 'self-evident' assumptions which are erroneous on closer inspection. The most prominent of them are the following.

(1) *The total wave function $\Psi_{tot}$ gives an exhaustive and correct description of the OCS.*

However, this is not – this function possesses the following pathological properties. On the one hand, the standard wave-packet approach dealing with the probability density $|\Psi_{tot}(x,t)|^2$ shows the inseparability of transmission and reflection at all stages of scattering. On the other hand, the Bohmian approach dealing with the nonintersecting lines of the corresponding probability flow density shows the reverse (see Section II K).

From the viewpoints of classical mechanics and probability theory all one-particle observables and characteristic times can be correctly introduced only for the subprocesses of the OCS. Indeed, the main peculiarity of the OCS is that each particle taking part in this process has two *alternative* possibilities – either to be transmitted or to be reflected by the barrier. As a consequence, the experimental study of this process implies making use of two detectors – one for transmitted particles and one for reflected ones. However, from the viewpoint of

classical probability theory, experimental data gathered with the help of these two macroscopic devices must not be handled jointly: as describing two mutually exclusive random events, these data are incompatible with each other and hence do not belong to the same Kolmogorovian probability space. From this it follows that the current interpretation of $|\Psi_{tot}(x,t)|^2$ as the probability density contradicts classical probability theory. QM, as a universal theory to be compatible with classical theories at the macro-level, must obey this requirement[59]. Thus, a true quantum-mechanical model of the OCS must provide the individual description of the transmission and reflection subprocesses at all stages of scattering.

(2) *Transmission and reflection are inextricably intertwined; assigning probabilities to interfering alternatives implies destruction of coherence between the alternatives, i.e., conversion of a pure state into a statistical mixture.*

However, our approach shows that this is not. For any potential barrier and initial state of a particle, the total wave function $\Psi_{tot}(x,t)$ can be uniquely decomposed into the sum of two subprocess wave functions $\psi_{tr}(x,t)$ and $\psi_{ref}(x,t)$. Yes, in the general case these quantum subprocesses cannot be considered as mutually exclusive, because the equality $<\psi_{tr}|\psi_{tr}> + <\psi_{ref}|\psi_{ref}>= 1$ is violated at the very stage of scattering. However, it is always satisfied at the first and final stages of scattering, when the distances between the wave packets and the potential barrier are asymptotically large. That is, by the example of the OCS, our approach demonstrates that quantum probabilities differ from classical ones at the micro-scales, but compatible with them at the macro-scales.

(3) The third assumption concerns *the character of the transmission (tunneling) dynamics in the barrier region* – in all the existing timekeeping procedures, including the one based on the group-velocity concept as well as all clock-based procedures, it is assumed as self-evident that this dynamics *is unitary like the quantum dynamics of the whole OCS.*

But if the tunneling dynamics was everywhere unitary, this subprocess could take place separately from the reflection one. At the same time, this is impossible for a non-resonant tunneling. As was shown in our approach, the transmission dynamics is 'asymptotically' unitary – the norm $<\psi_{tr}|\psi_{tr}>$ is the same at the initial and final stages of scattering. However, it is not preserved at the stage of interaction of the wave packet $\psi_{tr}(x,t)$ with the potential barrier. As a result, the group velocity to characterize the CM's dynamics cannot be used as a measure of the genuine velocity of transmitted (tunneled) particle in the barrier region. For the same reason the Larmor-clock procedure does not allow a direct measurement of the transmission time – because of changing the "number of particles" in the transmission subensemble at the midpoint of the barrier region the *average* spin of particles changes abruptly at this point, what is evident to be not associated with the the Larmor precession of the average spin of particles in the barrier region.

By our approach only the flow-velocity (or, energy-velocity, in CED) concept can be used to determine the velocity of transmitted particles in the barrier region as well as the time spent on average by these particles in this region. As regards the reflection subprocess, it is unitary. Thus, not only the flow-velocity concept but also the group-velocity concept can be used for timekeeping this subprocess in the barrier region. Unlike the transmission dwell time the reflection dwell time can be expressed through the final readings of the Larmor clock.

However, the transmission dwell time can be indirectly measured too. For this purpose one can use the presented definitions of the characteristic times for both subprocesses as well as the relationships between them.

So, the unaccustomed property of the subprocess wave function $\psi_{tr}$ at the joining point $x_c$ turns out to play the crucial role in the explanation why the superluminal *group* tunneling velocity, in the opaque limit, does not at all mean that the tunnelling velocity of *particles* is indeed superluminal in this case. At this place it is worthwhile to recall about the above referee comment where it is claimed that "the decomposition of the wave function into a transmitted and reflected portion at a point $x_c$ is not proven".

## VI. ACKNOWLEDGMENTS


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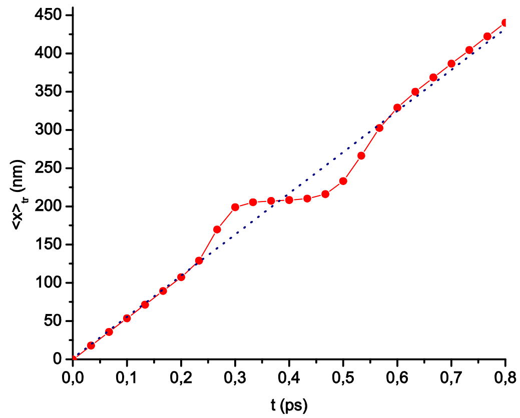


FIG. 1: The functions $X_{tr}(t)$ (solid line) and $X_{tr}^{in}(t)$ (dashed line).